Author: Lana Konnova

Situation 2: Reducing the Fraction

<u>Prompt</u> (given by Jeanne Shimizu)

A student found out that in case of the fraction $\frac{26}{65}$ the "cross - reducing" can lead to the right answer: $\frac{26}{65} = \frac{2}{5}$

The question was " At what condition does it work?"

<u>Commentary</u>

The foci consider several important aspects of the fractions. Focus 1 looks at students' possible misunderstanding of the place value of the numbers. Focus 2 looks at the students' possible misunderstanding of the symbolic representation of the product operation. Focus 3 examines the condition when the "cross - reducing" leads to the right answer in case of two-digital numbers in the numerator and denominator.

<u>Mathematical Foci</u>

Mathematical Focus 1

It is important for students to understand the place value of the numbers

A positional notation or place-value notation system is a number system in which each position is related to the next by a common ratio, called the base of that number system.
In the decimal number system base equals 10. Each place has a value of 10 times the place to its right.

Mathematical Focus 2

It is important for students to understand the symbolic representation of operation of multiplication.

1.Multiplication is written using the multiplication sign "×" between the terms
2.Multiplication is sometimes denoted by either a middle dot
3. In algebra, multiplication that involves variables is often written as a juxtaposition (e.g. xy for x times y or 5x for five times x). However, in the case of a juxtaposition numbers must be

surrounded by parentheses (e.g. 5(2) or (5)(2) for five times two). Otherwise, 52 represents the two-digital number.

Mathematical Focus 3

It is possible to find the conditions at which the "cross - reducing" leads to the right answer in case of two-digital numbers in the numerator and denominator.

One can represent the fraction that contains two digital numbers in the numerator and denominator using the variables m, n, and l (n=0,…9, m=0,…9, l=1,…9). In this case, the statement that the "cross - reducing" leads to the right answer, leads to the equation:

$$\frac{10m+n}{10n+l} = \frac{m}{l}, \text{ then} \qquad (*)$$

$$10\frac{n}{l} - \frac{n}{m} = 9$$

After solving this equation in natural numbers (n=0,…9, m=1,…9, l=1,…9) , and considering the case when m=0 in (*), one can get the answer to the problem (*)

1. n=m=l (n=1,…9, m=1,…9, l=1,…9)
2. n=6, m=2, l=5
3. n=6, m=1, l=4
4. n=0, m=0, l=1,…9