

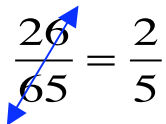
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## ***Situation 2: Reducing the Fraction***

**Prompt** (given by Jeanne Shimizu)

A student found out that in case of the fraction  $\frac{26}{65}$  the “cross - reducing” can lead to the right

answer:  $\frac{26}{65} = \frac{2}{5}$



The question was “ At what condition does it work?”

### **Commentary**

The foci consider several important aspects of the fractions. Focus 1 looks at students’ possible misunderstanding of the place value of the numbers. Focus 2 looks at the students’ possible misunderstanding of the symbolic representation of the product operation. Focus 3 examines the condition when the “cross - reducing” leads to the right answer in case of two-digit numbers in the numerator and denominator.

### **Mathematical Foci**

#### **Mathematical Focus 1**

*It is important for students to understand the place value of the numbers*

A positional notation or place-value notation system is a number system in which each position is related to the next by a common ratio, called the base of that number system.

In the decimal number system base equals 10. Each place has a value of 10 times the place to its right.

#### **Mathematical Focus 2**

*It is important for students to understand the symbolic representation of operation of multiplication.*

1. Multiplication is written using the multiplication sign "×" between the terms
2. Multiplication is sometimes denoted by either a middle dot
3. In algebra, multiplication that involves variables is often written as a juxtaposition (e.g. xy for x times y or 5x for five times x). However, in the case of a juxtaposition numbers must be

surrounded by parentheses (e.g. 5(2) or (5)(2) for five times two). Otherwise, 52 represents the two-digit number.

### Mathematical Focus 3

*It is possible to find the conditions at which the “cross - reducing” leads to the right answer in case of two-digit numbers in the numerator and denominator.*

One can represent the fraction that contains two digital numbers in the numerator and denominator using the variables  $m$ ,  $n$ , and  $l$  ( $n=0,\dots,9$ ,  $m=0,\dots,9$ ,  $l=1,\dots,9$ ). In this case, the statement that the “cross - reducing” leads to the right answer, leads to the equation:

$$\frac{10m+n}{10n+l} = \frac{m}{l}, \text{ then} \quad (*)$$

$$10\frac{n}{l} - \frac{n}{m} = 9$$

After solving this equation in natural numbers ( $n=0,\dots,9$ ,  $m=1,\dots,9$ ,  $l=1,\dots,9$ ), and considering the case when  $m=0$  in (\*), one can get the answer to the problem (\*)

1.  $n=m=l$  ( $n=1,\dots,9$ ,  $m=1,\dots,9$ ,  $l=1,\dots,9$ )
2.  $n=6$ ,  $m=2$ ,  $l=5$
3.  $n=6$ ,  $m=1$ ,  $l=4$
4.  $n=0$ ,  $m=0$ ,  $l=1,\dots,9$